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Return-volatility correlation in financial dynamics

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We investigate the return-volatility correlation both local and nonlocal in time with daily and minutely data of the German DAX and Chinese indices, and observe a leverage effect for the German DAX, while an antileverage effect for the Chinese indices. In the negative time direction, i.e., for the volatility-return correlation, an antileverage effect nonlocal in time is detected for both the German DAX and Chinese indices, although the duplicate local in time does not exist. A retarded volatility model may describe the asymmetric properties of the financial indices in the positive time direction.

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In recent years, the application of physical concepts and methods to economic and social science has attracted much attention of physicists. Based on large amounts of historical data, the dynamic behavior of financial indices or stock prices, etc. has been quantitatively analyzed [1–5]. Different models and theoretical approaches have been developed to describe the features of the financial dynamics [6–17].

The dynamics of a complex system is often characterized by dynamic fluctuations and time correlations. For the financial system, let us denote a financial index and the return of the index as $y(t')$ and $r(t')=\ln y(t'+1)-\ln y(t')$, respectively. Recent quantitative analyses show that the autocorrelation function of the volatility $|r(t')|$ decays by a power law, i.e., with a very long correlating time [2,3]. This explains large fluctuations of the volatilities [1–3,5]. On the other hand, the autocorrelation function of the returns decays exponentially with a negligibly small correlating time [2,3], and the movement of $r(t')$ can not be naively predicted from its history.

In order to further understand the dynamic evolution of $r(t')$, one then considers the return-volatility correlation function

$$L(t)=[\langle r(t')|r(t'+t)|^2\rangle-\langle r(t')\rangle\langle|r(t')|^2\rangle]/Z, \quad Z=\langle|r(t')|^2\rangle^2, \tag{1}$$

which quantifies the asymmetric volatility [18–20]. Here $\langle\cdots\rangle$ is an average over t'. In a recent work [4], $L(t)$ is computed with the daily data of several mature financial markets, and a negative $L(t)$, i.e., the so-called leverage effect, is observed up to weeks. The leverage effect indicates that a negative $r(t')$ induces a higher volatility, while a positive $r(t')$ may lead to stable stock prices [4,18–21]. In the negative time direction, i.e., for the volatility-return correlation, $L(t)$ fluctuates around zero [4].

Up to date, dynamic properties local in time are mainly concerned. The features of the financial dynamics are rather robust, and usually independent of specific markets. The motivation of this paper is twofold. On the one hand, the Chinese stock market is newly set up in 1990. Due to different social and economic systems in China, it is important to study the dynamic behavior of the Chinese market and to broaden our understanding of the financial dynamics. On the other hand, the dynamic behavior nonlocal in time is much attended in the real markets. For example, one tells that the market has been continuously unstable for a few weeks, etc. An investigation of the dynamic behavior nonlocal in time gains new insights into the dynamic mechanism in financial markets.

In this Rapid Communication, we study the return-volatility correlation both local and nonlocal in time for the German and Chinese markets. For a comparative study, we have collected the daily data of the German DAX from 1959 to 1999 and the minutely data from 1993 to 1997, and the daily data of the Shanghai Index and Shenzhen Index from 1990 or 1991 to 2003, and the minutely data from 1999 to 2003. The minutely data are recorded every minute in the German DAX, while every 5 min in Chinese indices. A working day is about 400 min in Germany while 240 min in China [22].

In Fig. 1(a), $-L(t)$ computed with the daily data is plotted. In the positive time direction, we observe a negative $L(t)$, i.e., a leverage effect, for the German DAX, while a positive $L(t)$ for both the Shanghai Index and Shenzhen Index. We name a positive return-volatility correlation "an antileverage effect." Fitting the data to an exponential form $L(t)=c\exp(-t/\tau)$, one obtains $\tau=15$ and 7 days days for the leverage and antileverage effects, respectively. Compared with the small correlating time (a few minutes) of the returns [2,3], both the leverage and antileverage effects are rather prominent. Here the leverage hypothesis [23,24], which assumes a negative return-volatility correlation, fails in the Chinese market. In the negative time direction, $L(t)$ fluctuates around zero for both the German and Chinese markets. This implies that $r(t')$ is not or very weakly correlated to volatilities in the past times. In this sense, the volatility feedback theory [25,26], which assumes a negative volatility-return correlation, does not straightforwardly apply to both the German and Chinese markets [4]. For $L(t)$, the financial dynamics is irreversible in time.

Usually, the leverage effect is considered to be a phenomenon at the daily time scale, and therefore only computed with the daily data [4]. To further confirm our findings, however, we analyze also the minutely data. Our minutely data of both the German DAX and Chinese indices are taken only for a few years. If the findings from the daily data may be

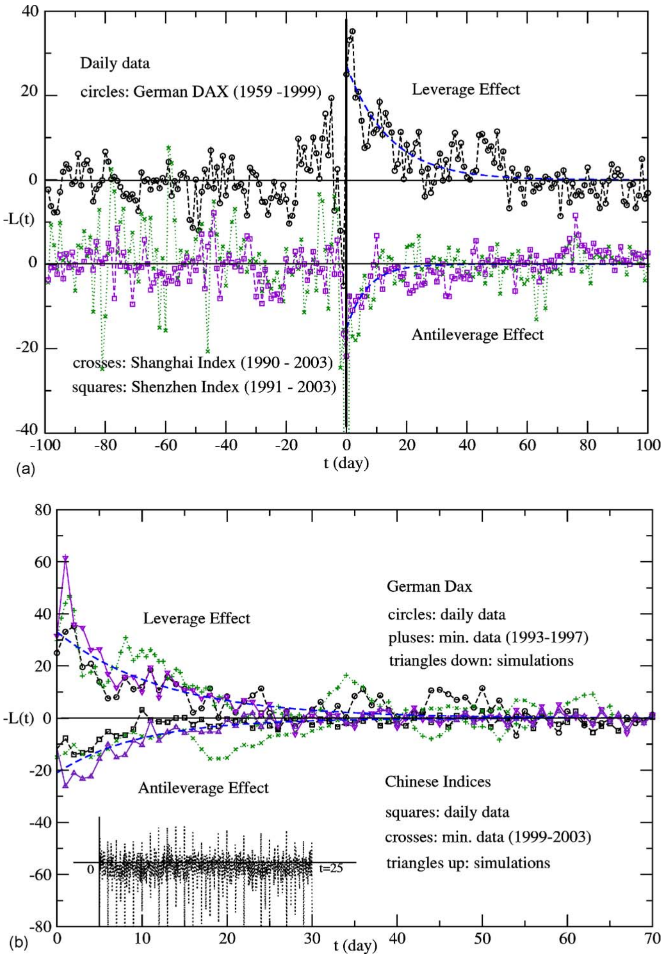

FIG. 1. (Color online) (a) $-L(t)$ computed with daily data. Dashed lines show an exponential fit $L(t)=c\exp(-t/\tau)$ with $(c,\tau)=(-27,15)$ and (15,7) for the leverage and antileverage effects, respectively. (b) $-L(t)$ computed with both daily and minutely data. The curves of the Chinese indices are from an average over the Shanghai index and Shenzhen index. Dashed lines are for guiding the eyes and with an exponential form $L(t)=c\exp(-t/\tau)$ with $(c,\tau)=(-33,13)$ and (21,10) for the leverage and antileverage effects, respectively.

also observed with the minutely data, we can safely conclude that the leverage and antileverage effects are indeed the features of the German DAX and Chinese indices, respectively. Obviously, as shown in the island of Fig. 1(b), $L(t)$ computed with the minutely data contains high-frequency fluctuations. To reveal the dynamic behavior of the slow mode, we average the data points in time windows of 4 days. Then the leverage and antileverage effects emerge, well consistent with those of the daily data. Since the dynamic behaviors of the Shanghai index and Shenzhen index look essentially the same, an average over two indices has been taken in Fig. 1(b).

Why do the German and Chinese markets exhibit different return-volatility correlations? Germany is a developed country. To some extent, people show risk aversion, and therefore, may be nervous in trading as the stock price is falling. This induces a higher volatility. When the price is rising, people feel safe and are inactive in trading. Thus, the stock price tends to be stable. This should be the social origin of the leverage effect. However, China just experiences the first stage of capitalism, and people are somewhat excessive speculative in the financial markets. Therefore, people rush for trading as the stock price increases. When the price drops, people stay inactive in trading and wait for rising up of the stock price. That explains the antileverage effect.

The correlation function $L(t)$ is an observable local in time, and the results in the negative time direction in Fig. 1(a) suggest that the volatility at a fixed time does not control the rising or falling of the price in the future. In the real markets, however, the dynamic behavior nonlocal in time can be important. Therefore, exploring the dynamic behavior nonlocal in time will deepen the understanding of the financial dynamics. Following this thought, we introduce a dynamic observable nonlocal in time to describe the return-volatility correlation.

For $r(t')>0$ (or <0), we define $P(t,+)$ [or $P(t,-)$] as the probability that the volatility has been always below $|r(t')|$ in a time t, i.e., $|r(t'+s)|<|r(t')|$ for all $s<t$. In practical computations, an average is taken over t'. $P(t,\pm)$ describe time correlations of the volatilities nonlocal in time, and similar dynamic behavior has been well studied in critical dynamics and turbulence [27–32]. For the returns $r(t')>0$ and $r(t')<0$, respectively, the probability distributions $P(t,+)$ and $P(t,-)$ measure how long the price would continuously remain stable. The difference between $P(t,+)$ and $P(t,-)$ indicates the effect of the return $r(t')$ on this time correlation of the volatilities nonlocal in time. Therefore, we define a return-volatility correlation function nonlocal in time,

$$NL(t)=[P(t,-)-P(t,+)]/Z, \quad Z=|P(1,-)-P(1,+)|. \tag{2}$$

A negative $NL(t)$ indicates that $|r(t'+t)|$ tends to be stable if $r(t')>0$, while fluctuates more if $r(t')<0$. This is a leverage effect. Similarly, a positive $NL(t)$ represents an antileverage effect. In Fig. 2(a), $-NL(t)$ is displayed. For the daily data, a leverage effect is clearly observed for the German DAX, while an antileverage effect for the Chinese indices. This is similar to the finding from $L(t)$. But $NL(t)$ of the German DAX now shows a power-law behavior rather than an exponential one, and remains nonzero within months. Fitting the data to the form

$$NL(t)=c/(t+a)^{\beta}, \tag{3}$$

one obtains an exponent $\beta=0.91(5)$. $NL(t)$ of the Chinese indices is a little fluctuating, but the antileverage effect can be identified up to two weeks. For the minutely data, both the German and Chinese markets exhibit a leverage effect. The antileverage effect nonlocal in time for the Chinese market is a phenomenon purely in a daily time scale. In Fig. 2(a), all the curves of $NL(t)$ could be fitted to the power law in Eq. (3).

To reveal the dynamic effect of the volatilities on the returns, we compute $NL(-t)$ along the negative time direction. Surprisingly, one finds an antileverage effect for both the German and Chinese markets in Fig. 2(b). $NL(-t)$ of the

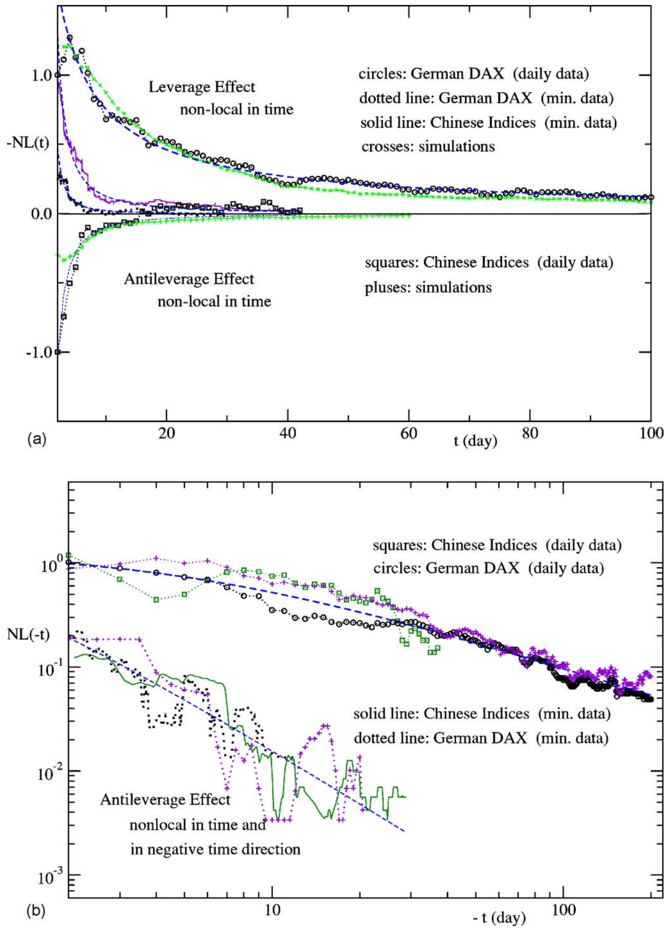

FIG. 2. (Color online) (a) $-NL(t)$ computed with both daily and minutely data. For clarity, the curves of the minutely data have been multiplied by a factor of 15. All the dashed lines obey a power law in Eq. (3). The exponent β extracted from the daily data of the German DAX is 0.91(5). For the minutely data of the German DAX, the daily data and minutely data of the Chinese indices, β is estimated to be 1.8, 1.5, and 1.5, respectively. (b) $NL(-t)$ plotted in a log-log scale. The curves of the minutely data have been multiplied by a factor of 15. The dashed lines obey a power law in Eq. (3). The exponent β extracted from the daily data of the German DAX is 0.88(5). The dashed line fitted to the minutely data is with an exponent $\beta=1.8$. For comparison, $-NL(t)$ of the daily and minutely data of the German DAX are shown with pluses.

daily data of the German DAX exhibits a power-law behavior up to months, and the fitted exponent $\beta=0.88(4)$. In fact, the curves of both the German DAX and Chinese indices obey the power law in Eq. (3) with a same exponent β, although the exponent β takes different values for the daily and minutely data. For comparison, we also plot $-NL(t)$ of the German DAX in Fig. 2(b). For $NL(t)$, the German DAX is reversible in time, while the Chinese indices are not.

Compared with $L(t)$, $NL(t)$ describes time correlations nonlocal in time, and is clearly nonzero in the negative time direction. This is significant. The antileverage effect in the negative time direction indicates that after the volatility $|r(t')|$ is continuously stable for a period of time, the price drops; otherwise, the price rises. In this sense, the dynamic behavior nonlocal in time also does not follow the volatility feedback assumption [4,25,26], but it should be reasonable. If the price keeps always unchanged, one feels little chance of benefit, and the demand of buying stocks drops. As a result, the stock prices fall. Such a phenomenon is universal for both the German and Chinese markets. All results above show that understanding the dynamic behavior nonlocal in time gains new insights into the financial dynamics. In fact, various dynamic correlation functions can be introduced for describing the dynamic properties nonlocal in time, and the detail of this kind will be presented elsewhere.

Finally, we introduce a retarded volatility model to simulate the leverage and antileverage effects. According to Ref. [4], the price change $dy(t')=y(t'+1)-y(t')$ depends not only on $y(t')$ but also on the prices in the past times,

$$dy(t') = \left[y(t') - \sum_{t=1}^{\infty} K(t) dy(t'-t) \right] \sigma(t') \epsilon(t'). \qquad (4)$$

Here $\epsilon(t')$ is a Gaussian white noise, $\sigma(t')$ is the volatility generated, e.g., by a dynamic herding model [10,16,17], and $K(t)$ represents the retarded effect of the prices, which should decay to zero within a certain time. Usually, $y(t')$ is with a not too small background, then $dy(t'-t)/y(t') \approx dy(t'-t)/y(t'-t)$. Keeping in mind that $r(t') \approx dy(t')/y(t')$, Eq. (4) leads to $r(t')=[1-\Sigma_{t=1}^{\infty} K(t) r(t'-t)]\sigma(t')\epsilon(t')$. Following the procedure in Ref. [4], one may approximately derive $L(t)=-2K(t)$ if $\sigma(t')$ is the order of 1. Taking $K(t) = \pm c \exp(-t/\tau)$, we numerically simulate the retarded volatility model and compute $L(t)$. A positive $K(t)$ induces a leverage effect, while a negative $K(t)$ leads to an antileverage effect. The results are shown in Fig. 1(b). In addition, the solution also reproduces the leverage and antileverage effects nonlocal in time in Fig. 2(a). A power-law behavior of $NL(t)$ is observed. However, the antileverage effect in the negative time direction cannot be described by the dynamics of Eq. (4). It remains a great challenge to understand fully the leverage and antileverage effects in financial markets.

In summary, with both the daily and minutely data of the German DAX and Chinese indices, we compute the return-volatility correlation functions both local and nonlocal in time, and find a leverage effect for the German DAX while an antileverage effect for the Chinese indices. In the negative time direction, i.e., for the volatility-return correlation functions, an antileverage effect nonlocal in time is detected for both the German DAX and Chinese indices, although the duplicate local in time does not exist. A retarded volatility model may describe the asymmetric properties of the financial indices in the positive time direction.

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